Geological Society of Malaysia Annual Geological Conference 2001
June 2–3 2001, Pangkor Island, Perak Darul Ridzuan, Malaysia

Spectral-Analysis-of-Surface-Waves method: An initial assessment and its potential use in geology

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Abstract: The Spectral-Analysis-of-Surface-Waves (SASW) method for profiling the subsurface non-destructively is discussed. The method assumes that the subsurface structures consist of a stack of horizontally homogeneous layers. Transient impact source on the ground surface is used to generate Rayleigh wave of different frequencies into the medium. From analysis of phase information for each frequency, the velocity of the waves is determined between two receivers. Initial results of the SASW measurements on flexible and rigid pavement systems are presented.

Abstrak: Kaedah SASW (Analysis-Spektrum-Gelombang-Permukaan) bagi mendapatkan profil di bawah permukaan secara tanpa memusnahkan adalah dibincang. Kaedah ini mengandaikan bahawa struktur di bawah permukaan adalah terdiri daripada lapisan mendatar yang homogen. Punca hentaman di permukaan menghasilkan gelombang Rayleigh berbagai frekuensi yang merambat di sepanjang permukaan. Analisis dari maklumat fasa bagi setiap frekuensi menghasilkan halaju gelombang yang bergerak di antara dua pengesan. Kertas ini membentangkan hasil awal pengukuran kaedah SASW yang dilakukan ke atas sistem turapan.

INTRODUCTION

Rayleigh waves were first described quantitatively by Lord Rayleigh in 1885. Since then the surface waves have been utilised by researchers in a number of application sectors. In geotechnical engineering sector, for instance, the use of Rayleigh waves is not new. Geotechnical engineers such as Terzaghi (1943) and Hvorslev (1949) were among the pioneers of surface wave geophysics. Jones (1958), on the other hand, used surface wave to assess materials under road pavements but his work was unsuccessful because of the insensitive recorders and signal receivers being used at that time.

With the advent of spectral analysis and portable computers in late 1970s, the traditional surface wave technique has revolutionised to the Spectral-Analysis-of-Surface-Waves (SASW) method. Over the past decade, the SASW method has attracted many engineers and has been utilised in different applications. These application areas include characterisation of foundation (Madshus and Westerdahl, 1990; Stokoe *et al.*, 1994), non-destructive evaluation and characterisation of pavement systems (Nazarian and Stokoe, 1984), evaluation of concrete structures (Rix *et al.*, 1990), and *in situ* determination of ground stiffness (Matthews *et al.*, 1996).

The SASW method is based on the analysis of the dispersive characteristic of Rayleigh waves in a non-homogeneous medium. The method is a non-destructive test method in which both the source and the receivers are located on the ground surface. No requirement for expensive boreholes, repeatability of the test, and a simple set-up and test procedure are among the advantages of this technique. Because of the dispersive characteristics of Rayleigh waves in a heterogeneous medium, the analysis of the frequency spectrum will produce valuable information about the inhomogeneity in the ground.

This paper describes SASW technique in terms of basic principles, instrumentation, field testing procedures, data reduction, and briefly presents some of the preliminary results of SASW test for rigid and flexible pavement systems.

PRINCIPLE OF THE SURFACE WAVE METHOD

Surface waves propagate along the surface of a half-space and their amplitude decreases rapidly with depth. The surface waves resulting from a vertical impact are primarily Rayleigh waves. Rayleigh waves propagate away from the impact along a cylindrical wave front near the surface of the medium, at a speed of approximately 90% of shear waves. Particle motion associated with Rayleigh waves is composed of both vertical and horizontal components, which when combined, form a retrograde ellipse close to the surface (Figure 1). However, with depth Rayleigh waves particle motion changes to pure vertical and finally, to a prograde ellipse.

In homogeneous, isotropic, elastic half-space, Rayleigh wave velocity does not vary with frequency. However, Rayleigh wave velocity varies with frequency in a layered medium when there is variation of stiffness with depth. This frequency dependency of surface wave velocity in a layered system is termed dispersion, and a plot of velocity versus frequency (or wavelength) is called dispersion curve. The dispersive characteristic of surface waves can be demonstrated by examining the phase velocity, which is defined as the velocity with which a seismic disturbance of a given frequency propagates in a medium.

Surface waves could be used to determine shear stiffness in soil and rocks. The theory of elasticity shows that the relationship between the characteristic velocity of shear waves, V_s and Rayleigh waves, V_r in an elastic medium is given by:

$$V_r = CV_s \qquad (1)$$

where C is a function of Poisson's ratio, n. The range of C is from 0.911 to 0.955 for the range of Poisson's ratio associated with most soils and rocks if anisotropy is ignored. The maximum error in G arising from an erroneous value of C is probably less than 10% (Nazarian, 1984).

TESTING PROCEDURE OF THE SASW METHOD

Field Procedure

Data is collected from the ground surface by generating Rayleigh waves at one point (the source), detecting them at two other points (receivers), and recording the signals for future analysis (the spectral analyser), as shown schematically in Figure 2. In a SASW test, two receivers are placed on the surface, and a hammer is used to generate the wave energy. Several sets of tests with different receiver spacing are typically required to sample different layers. Short receiver spacing with high frequencies (short wavelengths) are used to sample shallow layers, while long receiver spacing with low frequencies (long wavelengths) are used in sampling the deep materials (Figure 3). Two profiles, a forward and a reverse profile, are normally obtained in SASW measurements where the accessible surface is struck by a hammer on two opposite sides of the receivers, as shown in Figure 2. These profiles, when averaged, can minimise the effect of internal phase shift between receivers (Nazarian, 1984).

A dynamic signal analyser is used to collect and transform the receiver outputs to the frequency domain. Two functions in the frequency domain are of great importance in SASW tests: (1) the coherence function, and (2) the phase information of the cross power spectrum between the two receivers. The coherence function is a measure of the degree by which input and output signals are linearly correlated. The value close to one is an index of good correlation and hence the recorded signals can be considered genuine and unaffected by ambient noise (Figure 4). Therefore, data collected in the field can be conveniently checked, and the test can be repeated if necessary. Typically, the average of five individual tests is used to determine the spectral functions.

The cross power spectrum is used to obtain the relative phase shift between two signals (two-channel recorder) at each frequency in the range of frequencies excited in the SASW test (Figures 5). This phase shift can be translated into travel time.

Data Reduction

Based on the dispersive characteristics of Rayleigh waves in non-homogeneous media, a plot of the wavelength versus phase velocity is known as the dispersion curve. The experimental dispersion curve may be developed from phase information of the cross power spectrum at frequency ranges satisfying the coherence criterion. The range of frequencies that is contaminated can be identified and rejected during data reduction. However, using coherence alone may not always lead to good data. Instead, the parallel use of observation and judgement of both the coherence and the phase of the cross spectrum leads to better results. For a travel time equal to the period of the wave, the phase different is 360°. Thus, for each frequency the travel time between receivers can be calculated by:

$$t(f) = \frac{f(f)}{(360f)} \qquad (2)$$

where f is frequency, $t(f)$ is travel time of the given frequency, and $f(f)$ is the phase difference in degrees of the given frequency. The distance between the receiver (x) is a known parameter. Therefore, Rayleigh wave velocity at a given frequency is simply calculated by:

$$V_r(f) = \frac{V_r(f)}{t(f)} \qquad (3)$$

and the corresponding wavelength of the Rayleigh wave is equal to:

$$L_r(f) = \frac{V_r(f)}{f} \qquad (4)$$

By repeating the procedure outlined by Equation (2) through (4) for every frequency, the Rayleigh wave velocity corresponding to each wavelength is evaluated and the dispersion curve is determined (Figure 6). It is important to note that velocities obtained from the experimental dispersion curve are not actual Rayleigh wave velocities, but rather apparent or phase velocities. The existence of a layer with a higher or lower velocity at the surface of a medium affects the measurement of the velocities for the underlying layers. Thus, a method for evaluating actual Rayleigh wave velocities from apparent Rayleigh wave velocities is necessary in SASW test.

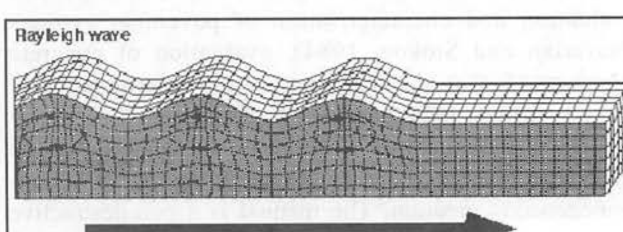

Figure 1. Elastic deformations and ground particle motion associated with the propagation of Rayleigh waves. Modified from Bolt (1978).

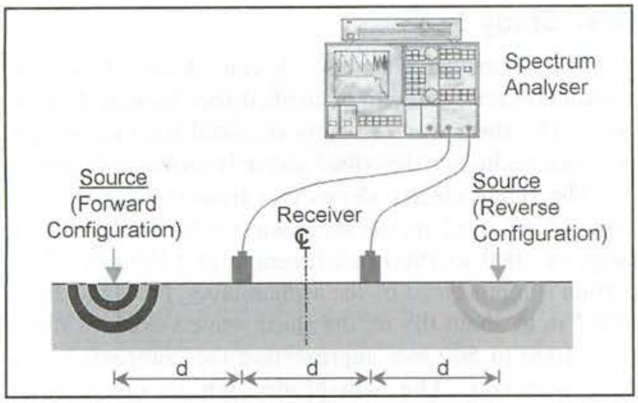

Figure 2. Basic configuration of equipment for SASW measurements.

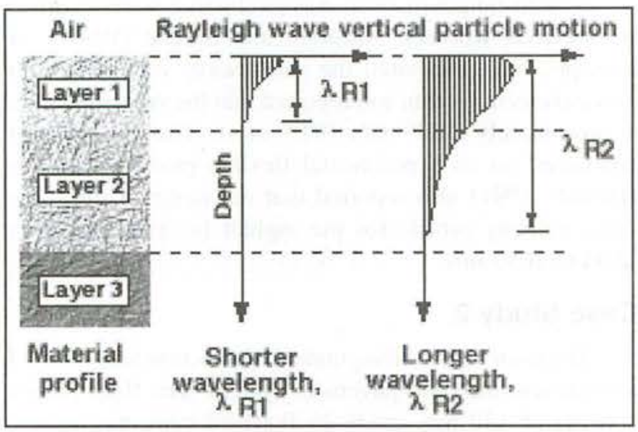

Figure 3. Variation of vertical particle motion for Rayleigh waves with different wavelengths.

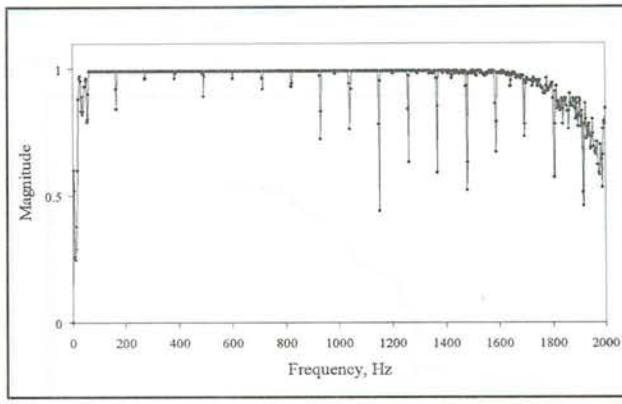

Figure 4. Coherence function of spectral analysis measurement on asphalt pavement for a 100 cm receiver spacing.

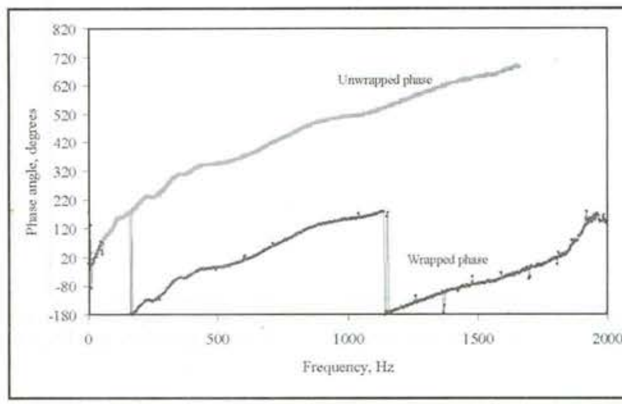

Figure 5. A typical phase plot of the cross power spectrum from SASW measurement on asphalt pavement for a 100 cm receiver spacing.

Inversion of Dispersion Curve

The process of determining the actual propagation velocities at different depth (velocity profile) from the experimental dispersion curve is known as the inversion (back-calculation) of the Rayleigh wave dispersion curve. There are basically two inversion processes, a simple and a refined one.

(i) Simple Inversion

The simple inversion process is based on an earlier inversion of the steady-state vibration technique (Richart *et al.*, 1970). The inversion is done by re-scaling the phase velocity axis to get the shear wave velocity, and the wavelength axis to get the depth. This procedure was used in the preliminary investigation of the SASW method. The approximate inversion technique is performed by assigning the shear wave velocity equals to 1.1 times the phase velocity and the depth is 0.33 times the wavelength (Heisey *et al.*, 1982). It provides satisfactory results for geotechnical sites where the shear modulus smoothly increases with depth (Matthews *et al.*, 1996). However, in cases where there is a large contrast in shear wave velocities, this simple method can lead to erroneous results. This is especially true in a pavement system in which the stiffness of the materials can differ greatly.

(ii) Refined inversion

A more reliable inversion procedure uses stress-wave propagation theory. The propagation theory models a theoretical dispersion curve, which is compared with experimental dispersion curve. There are several modelling approaches available for SASW applications. Among them are: (1) the transfer matrix method (Thomson, 1950; Haskell, 1953); (2) the dynamic stiffness matrix method (Kauser and Roesset, 1981); and (3) the finite difference method (Hossain and Drnevich, 1989). The transfer and stiffness matrix methods provide exact formulation as compared to the other methods (Ganji *et al.*, 1998). Although the approach of the inversion methods can be different, however, all the methods assume that the profile consists of a set of homogeneous layers extending to infinity in the horizontal direction. The last layer is usually considered as a homogeneous half-space.

Joh (1996) has suggested two methodologies for the evaluation of a stiffness profile: the iterative forward modelling analysis and the inversion analysis. The iterative forward modelling analysis requires an initial guess of the stiffness profile. To each layer a thickness, a shear wave velocity, a Poisson's ratio (or compression wave velocity), and a mass density are assigned. Many studies have been carried on about the influence of each one of these

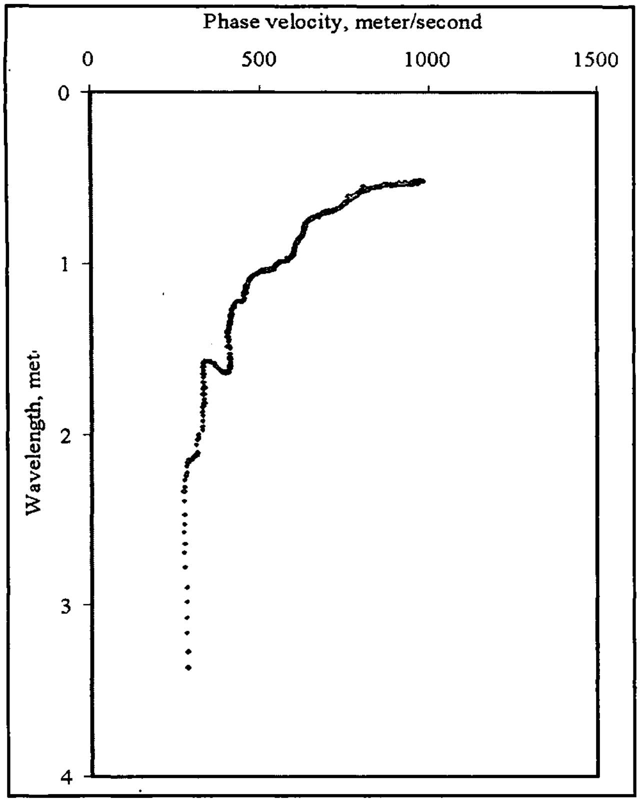

Figure 6. Dispersion curve constructed from record shown in Figure 5.

parameters and the general conclusion is that the influence of density and Poisson number is negligible (Nazarian, 1984). Therefore, they can be estimated on the basis of experience without any sensible effects on the final results. Based on the initial profile, a theoretical dispersion curve is calculated using stress wave propagation theory. The theoretical dispersion curve is compared with the experimental dispersion curve. If the two dispersion curves do not match, the initial profile (number of layers, layer thickness, shear wave velocity, or any combination) is adjusted, and another theoretical dispersion curve is calculated. The trial-and-error procedure is repeated until the two curves match, then the associated assumed profile is considered to be the real profile.

The inversion analysis is an automated forward modelling analysis. The stiffness profile is adjusted by an optimisation technique. Therefore, the optimum path from the initial guess to the final solution is found by the inversion analysis. There are a few inversion procedures suitable for SASW measurements.

RESULTS AND DISCUSSION

To demonstrate the versatility of the method a number of SASW measurements were carried out on flexible and rigid pavements, both at highways and the floor of a building. These sites are selected as their profile records are well documented, therefore, the accuracy of the inverted profiles can be estimated by direct comparison with the actual profile.

Case Study 1

The flexible pavement profile consisted of 200 mm of asphalt concrete, 450 mm of crushed-rock base, and a sub-grade. The shear wave velocity obtained from the simple inversion technique described above is shown in Figure 6.

The figure clearly shows that from the surface to a depth of about 0.2 m, the shear wave velocities are in the range of 1000 to 1900 m/s (average of 1450 m/s). This portion is represented by the asphalt layer. From the depth of 0.2 m to about 0.5 m, the shear wave velocities range from 1000 to 500 m/s, representing the compacted rock base material. The sub-grade, which consists of metasediments, is represented by the shear wave velocities of about 250 m/s.

Similar ranges of shear wave velocities were recorded by previous workers. Nazarian and Stokoe (1984), for example, have measured the shear wave velocities on a new pavement section and reported that the velocities were approximately 3000 ft/s (915 m/s). The SASW test conducted on an experimental flexible pavement by Al-Hunaidi (1991) also reported that the range of the shear wave velocity profile for the asphalt layer ranged from 1100 to 1650 m/s.

Case Study 2

The floor of a building under construction was selected to represent the rigid pavement system. The floor profile consists of 150 mm grade 25 Portland cement concrete, underlain by 50 mm sand, 150 mm hard core material, and the sub-grade.

As expected, the shear wave velocities for the cement concrete are slightly higher than those for the asphalt concrete. The top layer has shear wave velocity of 2000 to 2250 m/s and it extended to a depth of approximately 0.15 m. The compacted rock base material underneath has shear wave velocities range from 500 to 1500 m/s. However, the shear wave velocities for the sub-grade is higher compared with those for the above profile (asphalt pavement) and range from 250 to 400 m/s.

The result is comparable with the data from Nazarian and Stokoe (1984) for SASW test conducted on the airport rigid pavement. They found that the shear wave velocity for the concrete was about 9000 ft/s (2743 m/s), which is slightly higher because, as expected, higher quality of concrete is required for the runaway.

CONCLUSIONS

The SASW method is a relatively new seismic technique and has been under continuous development during the last few years, in particular the data inversion analysis. The method avoids the problems associated with borehole based methods. The non-destructive and non-invasive nature of the testing procedure avoids sampling disturbance and unrepresentative sampling. The most important characteristic of SASW method is that it can be

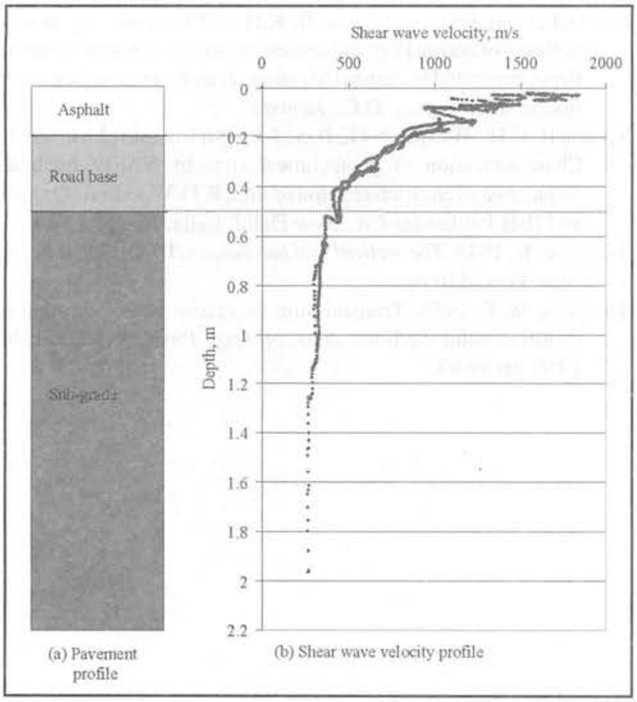

Figure 7. Comparison between the actual inverted shear wave velocity profiles from SASW measurements on a flexible pavement site.

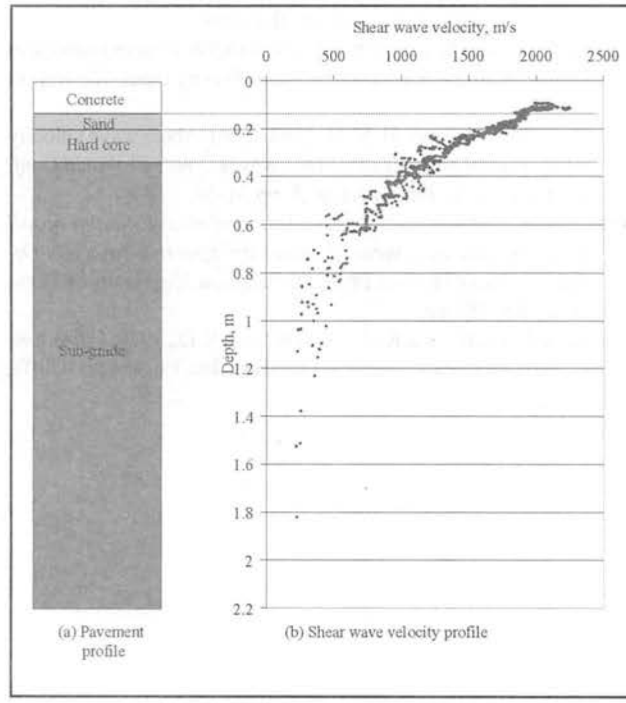

Figure 8. Comparison between the actual inverted shear wave velocity profiles from SASW measurements on a rigid pavement site.

used for irregular profiles, including a profile with a softer layer trapped between stiffer layers, a profile with a stiffer layer sandwiched between two softer layers, and a profile with a softer layer at depths. The profile presented in this paper, however, has not shown any of the above phenomenons as the bedrock is expected to be much deeper.

The SASW methods should be able to be extended to studies on geological formations that have distinct layering that model the half-space, such as layered sediments and metasediments. Miller and Xia (1999), for instance, have demonstrated the use of Multi-channel Analysis of Surface Waves (MASW) to map bedrock surface at depths of 6 to 23 ft and identified fracture zones within bedrock at a site in Kansas.

ACKNOWLEDGEMENTS

This work was supported by IRPA grant No. 02-02-02-0010 from Ministry of Science, Technology and the Environment. This support is gratefully acknowledged. The authors wish to thank Mr. Tajul Arus and Mr. Hamid at Geology Programme, and Mr. Ghani and Mr. Sulaiman at Faculty of Engineering for their assistance. Thanks also extended to Lakam, Rahman and Hafizal, Research Assistances at Geology Programme, for their help in the field.

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